Network Structure and Measures

After having discussed the basic building blocks of networks in detail, let us now deal with ways to capture and describe the structure of networks. The following measures are available for these tasks:

- Connectivity (Beta-Index)
- Diameter of a graph
- Accessibility of nodes and places
- Centrality / location in the network
- Hierarchies in trees

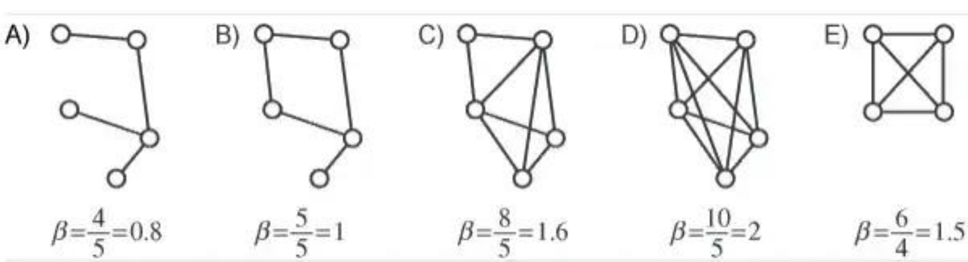
For most of these measures we will present one unweighted and one weighted (metric) case.

Connectivity (Beta index)

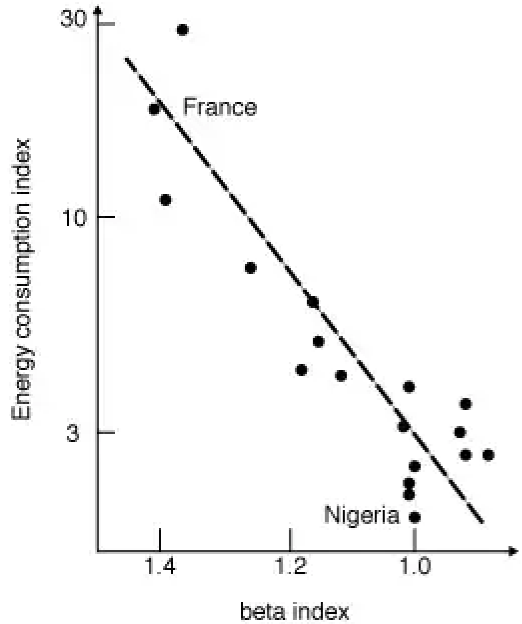
The simplest measure of the degree of connectivity of a graph is given by the Beta index (β). It measures the density of connections and is defined as:

$$\beta = \frac{E}{V}$$

where E is the total number of edges and V is the total number of vertices in the network.



In the figure above, the number of vertices remains constant in A, B, C and D, while the number of connecting edges is progressively increased from four to ten (until the graph is complete). As the number of edges increases, the connectivity between the vertices rises and the Beta index changes progressively from 0.8 to 2. Values for the index start at zero and are open-ended, with values below one indicating trees and disconnected graphs (A), and values of one indicating a network which has only one circuit (B). Thus, the larger the index, the higher the density. With the help of this index, regional disparities can be described, for example. In the figure below, the railway networks of selected countries are compared to general economic development (using the energy consumption-index of the 1960s). Energy consumption is plotted on the y-axis and the Beta index on the x-axis. Where connectivity is high, the economic development is high as well

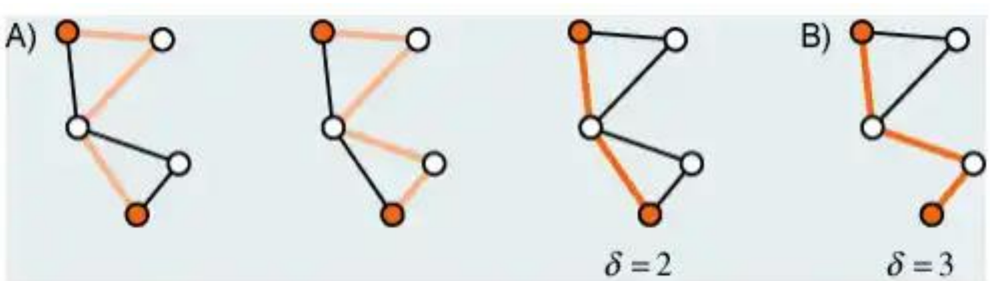


Diameter of a graph

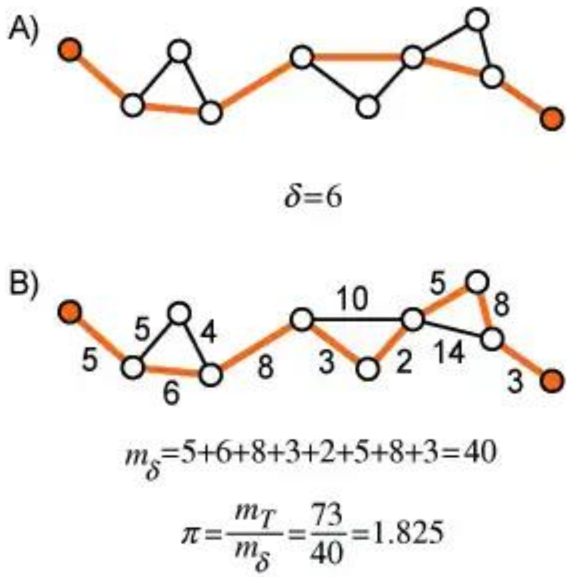
Another measure for the structure of a graph is its diameter. Diameter δ is an index measuring the topological length or extent of a graph by counting the number of edges in the shortest path between the most distant vertices. It is:

$$\delta = \max_{ij} \{s(i,j)\}$$

where s(i, j) is the number of edges in the shortest path from vertex i to vertex j. With this formula, first, all the shortest paths between all the vertices are searched; then, the longest path is chosen. This measure therefore describes the longest shortest path between two random vertices of a graph.



The first two figures in graph A show possible paths but not the shortest paths. The third figure and figure B show the longest shortest path.



In addition to the purely topological application, actual track lengths or any other weight (e.g. travel time) can be assigned to the edges. This suggests a more complex measurement based on the metric of the network. The resulting index is $\pi = mT/m\delta$, where mT is the total mileage of the network and mδ is the total mileage of the network's diameter. The higher π is, the denser the network.

Accessibility of vertices and places

A frequent type of analysis in transport networks is the investigation of the accessibility of certain traffic nodes and the developed areas around them. A measure of accessibility can be determined by the method shown in the animation. The accessibility of a vertex i is calculated by:

$$E_i = \sum_{j=1, j\neq i}^{v} n(i,j)$$

where v = the number of vertices in the network and n (i, j) = the shortest node distance (i.e. number of nodes along a path) between vertex i and vertex j. Therefore, for each node i the sum of all the shortest node distances n(i, j) are calculated, which can efficiently be done with a matrix. The node distance between two nodes i and j is the number of intermediate nodes. For every node the sum is formed. The higher the sum (node A), the lower the accessibility and the lower the sum (node C), the better the accessibility. The importance of the node distance lies in the fact that nodes may also be transfer stations, transfer points for goods, or subway stations. Therefore, a large node distance hinders travel through the network.

As with the diameter of a network, a weighted edge distance can also be used along with the pure topological node distance. Examples of possible weighting factors are: distance in miles or travel time as well as transportation cost. For this weighted measure, however, the edge distance is used and not the node distance.

$$E_i = \sum_{j=1}^{e} s(i, j)$$

where e is the number of edges and s(i, j) the shortest weighted path between two nodes.

Centrality / Location in the network

The first measure of centrality was developed by König in 1936 and is called the König numberKi. Let s(i, j) denote the number of edges in the shortest path from vertex i to vertex j. Then the König number for vertex i is defined as:

$$K_i = \max_{j \neq i}\{s(i, j)\}$$

where s(i, j) is the shortest edge distance between vertex i and vertex j. Therefore, Ki is the longest shortest path originating from vertex i. It is a measure of topological distance in terms of edges and suggests that vertices with a low König numbers occupy a central place in the network.

Hierarchies in trees

In quantitative geomorphology, more specificall in the field of fluvial morphology, different methods for structuring and order of hierarchical stream networks have been developed. Thus, different networks can be compared with each other (e.g. due to the highest occurence order or the relative frequencies of the unique levels), and sub-catchments can be segregated easily. Of the four ordering schemes in the following figure, only three are topologically defined. The Horton scheme is the only one that takes the metric component into account as well.

Calculating the strahler number, we start with the outermost branches of the tree. The ordering value of 1 is assigned to those segments of the stream. When two streams with the same order come together, they form a stream with their order value plus one.